Interaction of an Intense Electromagnetic Pulse with a Plasma

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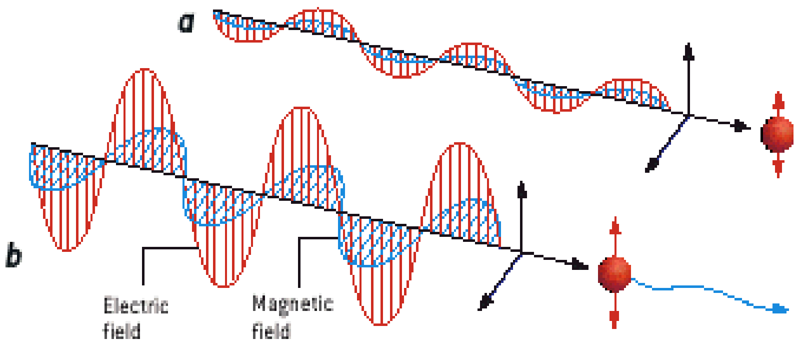


$\leftarrow \; v \times B$ Force is negligible

Electrons are non-relativistic $v << c$

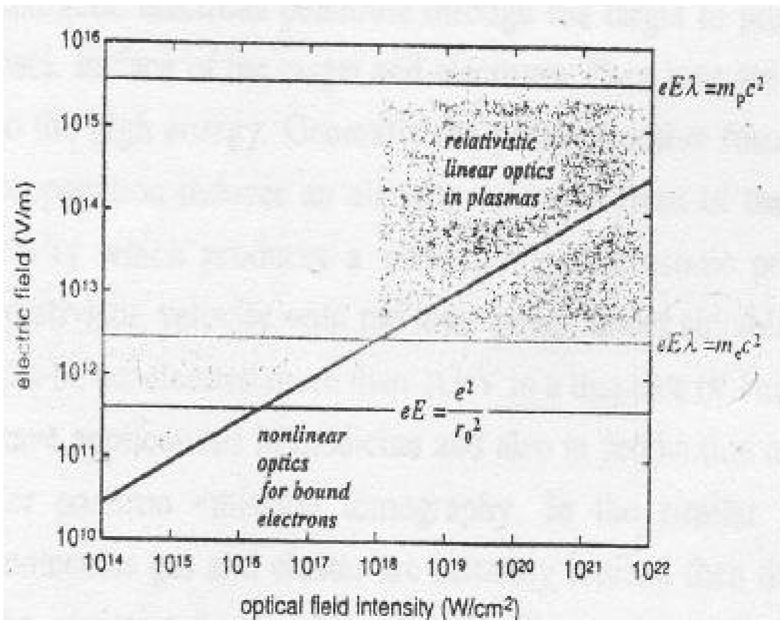
No propagation when $\omega < \omega_{pe}$

$\leftarrow \; v \times B$ Force is important

Electrons are relativistic $v \sim c$, relativistic mass correction become important.

$$\omega_{eff} = \omega_{pe}/\gamma^{1/2} < \omega_{pe}$$

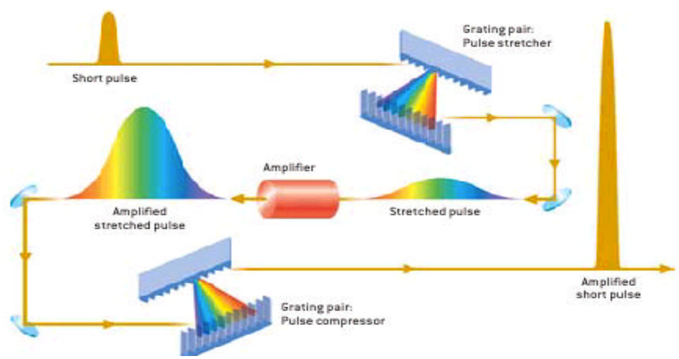
Intensity, $I \geq 10^{18} \; W/cm^2$



Sources of intense EM pulses:

Astrophysical systems like Pulsars (Neutron starts).

New generation laser pulses (CPA technique)

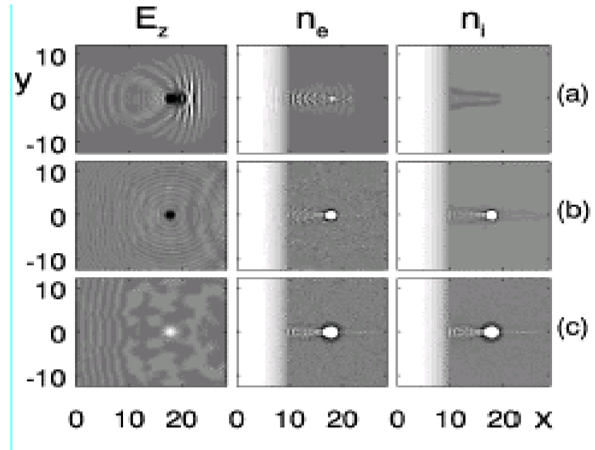


Light wave $\Longrightarrow$ Plasma $\Longrightarrow$ Variety of nonlinear phenomena

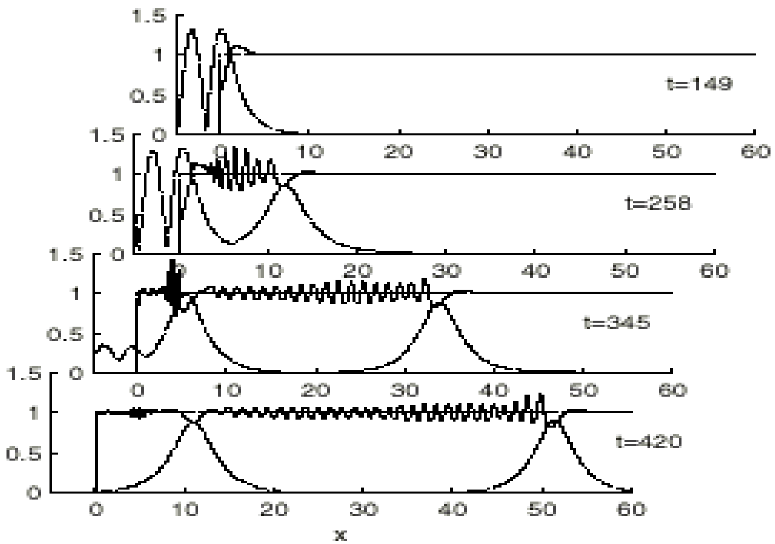
Envelope Solitons

- Modulated, coupled entities of light wave & plasma wave.
- Ponderomotive force of the light pulse become important..

- *PIC simulation*

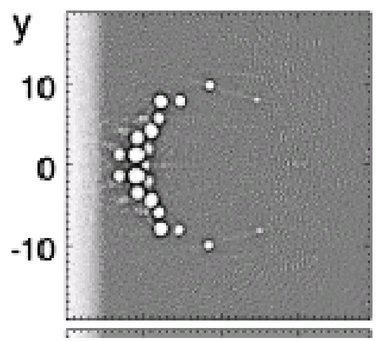


Fluid simulation



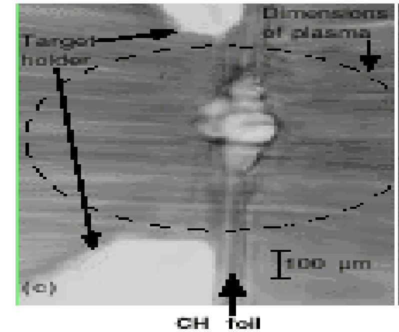
Tushentsov et al, 2001

Soliton necklace



Bulanov et al, 2002

Experiments



Borghesi et al, 2002

Applications

- Generation of X-ray pulses, giant magnetic pulses, hot electron channels.
- Acceleration of electrons, photons.
- Mimic astrophysical phenomena in the Laboratory which gives clues about the underlying physics.
- Fast ignition method of laser fusion: To heat the core of the fuel target in a short time using hot electron or energetic ions.

Overview of the thesis

- Introduction
- Envelope solitons in a cold plasma
- Warm plasma effects on slowly propagating weakly relativistic envelope solitons.
- Warm plasma effects on arbitrary amplitude structures.
- Envelope solitons in an electron-positron-ion admixture plasma.
- New classes of envelope solitons.
- Conclusions

2. Cold Plasma solitons

1-D model equations

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_{e\|}) = 0,$$

$$\left[\frac{\partial}{\partial t} + v_{e\|}\frac{\partial}{\partial x}\right](\gamma_e v_{e\|}) = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma_e}\frac{\partial}{\partial x}|A|^2,$$

$$v_{e\perp} = \frac{A}{\gamma_e},$$

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_{i\|}) = 0,$$

$$\left[\frac{\partial}{\partial t} + v_{i\|}\frac{\partial}{\partial x}\right](\gamma_i v_{i\|}) = -a\frac{\partial \phi}{\partial x} - a\frac{1}{2\gamma_i}\frac{\partial}{\partial x}|A|^2,$$

$$v_{i\perp} = -a\frac{A}{\gamma_i},$$

$$\frac{\partial^2 \phi}{\partial x^2} = n_e - n_i,$$

$$\frac{\partial^2 A}{\partial x^2} - \frac{\partial^2 A}{\partial t^2} = n_e v_{e\perp} - n_i v_{i\perp}.$$

New Variables: $\xi = x - \beta t, \quad t = \tau$

Solitons are stationary .

For circularly polarized light:

$$A = f(\xi)(\hat{y} + i\hat{z})e^{-i\lambda\tau} + c.c.$$

$$f(\xi) = R(\xi)\exp[i\theta(\xi)].$$

$R(\xi)$ is the amplitude; $\theta(\xi)$ phase of the pulse.

$$\gamma_e(1 - \beta v_{e\|}) - \phi = 1,$$

$$\gamma_i(1 - \beta v_{i\|}) + a\phi = 1,$$

$$n_e = \frac{\beta}{\beta - v_{e\|}},$$

$$n_i = \frac{\beta}{\beta - v_{i\|}},$$

$$\phi'' = n_e - n_i,$$

$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - \frac{n_e}{\gamma_e} - a\frac{n_i}{\gamma_i}\right] = 0.$$

(β *is the group velocity,* λ *is the eigenvalue which determines frequency*)

Bright solitons :

Light wave amplitude is maximum in the center and minimum or zero at infinity.

Single peak : **single peak in R & single hump in** ϕ

Multi-peak : **Multiple peaks in R & single hump in** ϕ

Boundary conditions :

$$R = 0, R' = 0 \text{ as } |\xi| \to \infty$$

Dark solitons :

Light wave intensity is minimum in the center and maximum infinity.

Earlier works on envelope solitons

- **Cold Plasma** :

® ***Kozlov et al, (1979)*** **: E-I plasma.**

Single peak : **quasineutral limit, continuous eigenvlaues, limit on group speed:** $\beta < \sqrt{m_e / m_i}$

Multi-peak : **discrete eigenvalues, limit on frequency exist.**

® ***Kaw et al (1992)*** : **Special case -- Ions are immobile and for** $\beta \rightarrow 1$

Single peak : **Perturbative analysis, continuous eigenvalues**

Multi-peak case : **Physical interpretation.**

Application : **Particle acceleration**

® ***Kuehl et al (1993), Dimant et al (1998) :*** **Complemented Kaw et al**

Single peak solitons : **discrete spectrum, wake exist, continuum approximation is valid for practical purposes.**

® *Esirkepov et al (1997)* : **Stationary solutions,** $\beta = 0$

Single peak solitons : Continuous specturm, arbitrary amplitudes.

Limit on frequency and hence on amplitude exist :

Physical constraint that electron density in the central region of the pulse should be non-negative.

® *Farina et al (2001)* : **Revisited Kozlov et al's case.**

Single peak solitons : Do not exist, no continuous transition from stationary structures to moving ones.

Multi-peak solutions : Transcritical limit is due to soliton breaking by ion dynamics.

Key issues : 1. Does single peak soliton exist in an overdense plasma?

2. What exactly is the nature of the transistion for single peak soliton?

3. Can one make some understanding for the multi-peak structure?

Simplified Equations (a=0)

$$\phi'' = \frac{1}{1-\beta^2}\left[\frac{\beta(1+\phi)}{\sqrt{(1+\phi)^2-(1-\beta^2)(1+R^2)}} - 1\right],$$

$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - \frac{\beta}{\sqrt{(1+\phi)^2-(1-\beta^2)(1+R^2)}}\right] = 0.$$

Constant of Motion :

$$K = \frac{R'^2}{2} - \frac{\phi'^2}{2(1-\beta^2)} + V(R,\phi),$$

where,

$$V(R,\phi) = \frac{\lambda^2}{(1-\beta^2)^2}\frac{R^2}{2} - \frac{\phi}{1-\beta^2} - \frac{\beta[\beta(1+\phi)+\sqrt{(1+\phi)^2-(1-\beta^2)(1+R^2)}]}{(1-\beta^2)^2}.$$

Single peak solitons (Immobile ions, a=0)

- **Stationary entity**

(Esirkepov et al, 1998)

$$\beta = 0, \quad v_e = 0$$

$$\phi = \sqrt{1+R^2} - 1$$

$$R = \frac{2\sqrt{1-\lambda^2}\cosh(\sqrt{1-\lambda^2}\xi)}{\cosh^2(\sqrt{1-\lambda^2}\xi) + \lambda^2 - 1}.$$

$$1 \le \lambda \le \sqrt{2/3}$$

- **Moving structures**

(kaw et al, 1992)

$$\beta \neq 0$$

$$\phi \approx \sqrt{1+R^2} - 1$$

$$n_e \approx (\sqrt{1+R^2})'' + 1$$

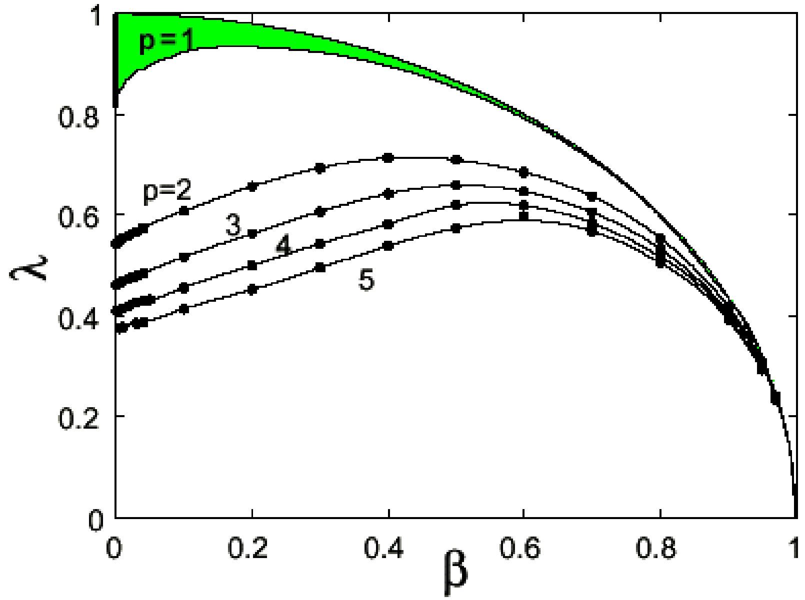
$$R = \sqrt{\frac{4(1-\beta^2)(1-\beta^2-\lambda^2)}{4\lambda^2-(1-\beta^2)(3+\beta^2)}}\,\mathrm{sech}\left[\frac{\sqrt{1-\beta^2-\lambda^2}}{1-\beta^2}\xi\right].$$

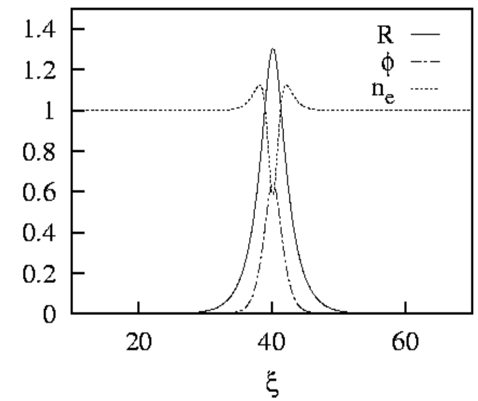
Complete evacuation of electron gives lower limit.

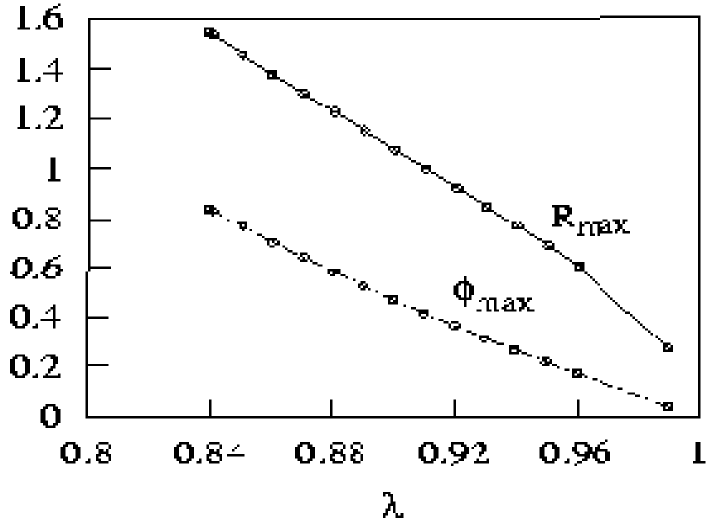
$$\beta \to 0, \quad \lambda \to 1$$

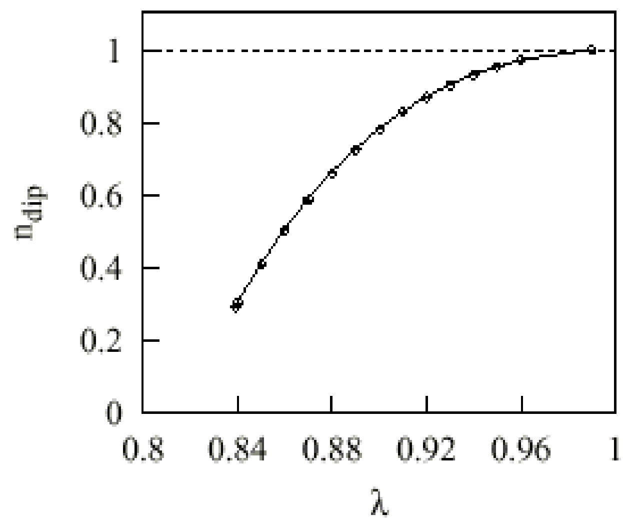
Moving single peak soliton smoothly connects to stationary solution.

Spectral Diagram

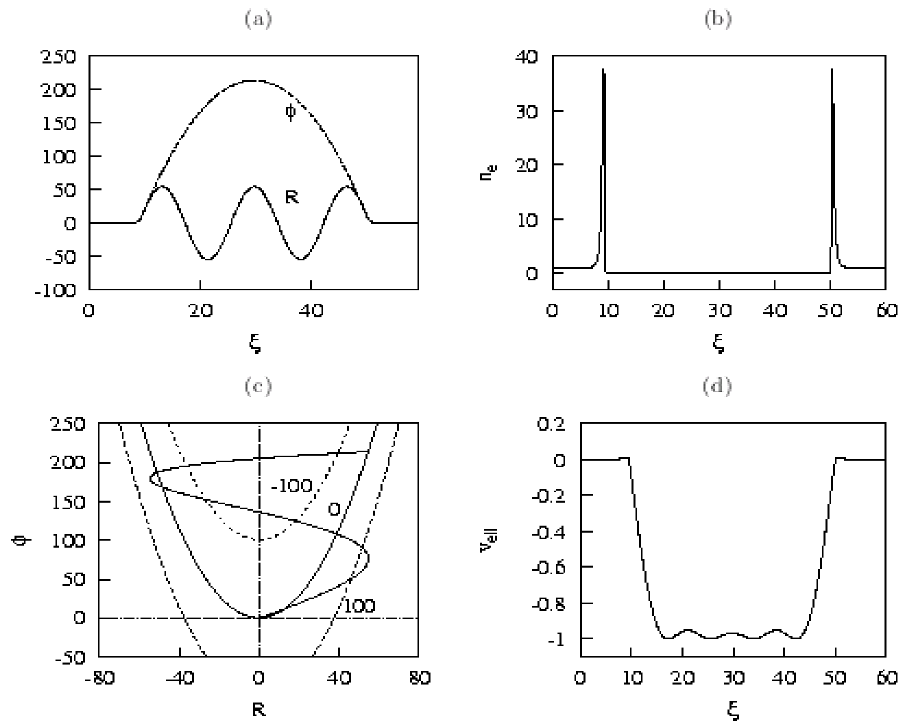








Multi-peak solitons



In the central region

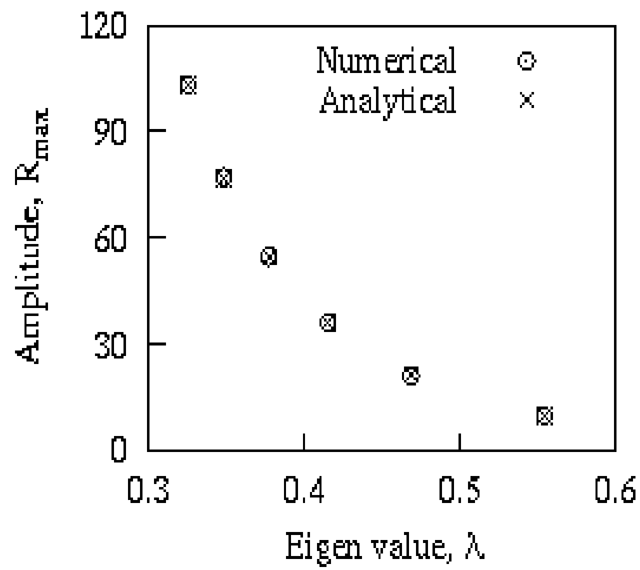
$$v_e \;\rightarrow\; -1, \quad \gamma_e \;\rightarrow\; 0\,,$$

$$\Rightarrow n_e \;\rightarrow \beta \;/\; 1 + \beta$$

$$\phi \approx \phi_{max} - \xi^2/2.$$

$$R = R_{max} \sin\left[\lambda\xi/(1-\beta^2)\right].$$

$$R_{max} = \frac{\pi p}{2\lambda^2}.$$



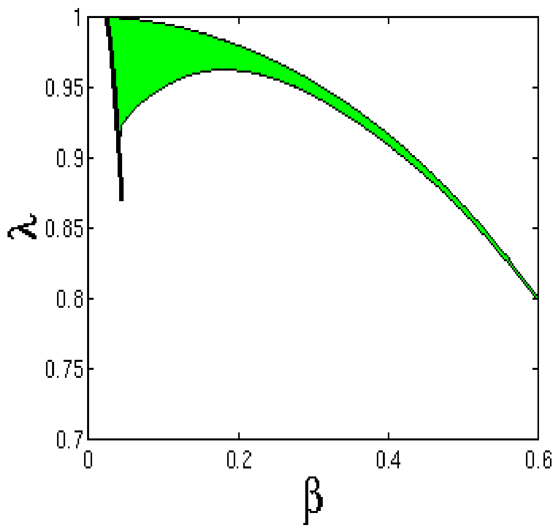
Mobile Ions

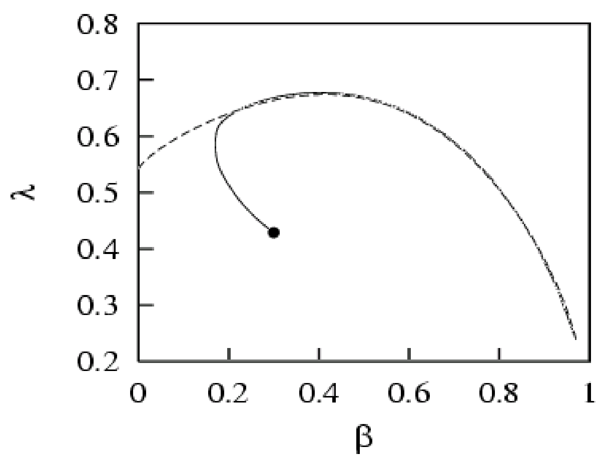
$$R(\xi)=\sqrt{\frac{1-\lambda^2-\beta^2}{(1-\beta^2)(\beta^2-a)}}\operatorname{sech}\left(\frac{\sqrt{1-\lambda^2-\beta^2}}{1-\beta^2}\right)$$

Kozlov et al, (1979) &

Farina et al, (2001)

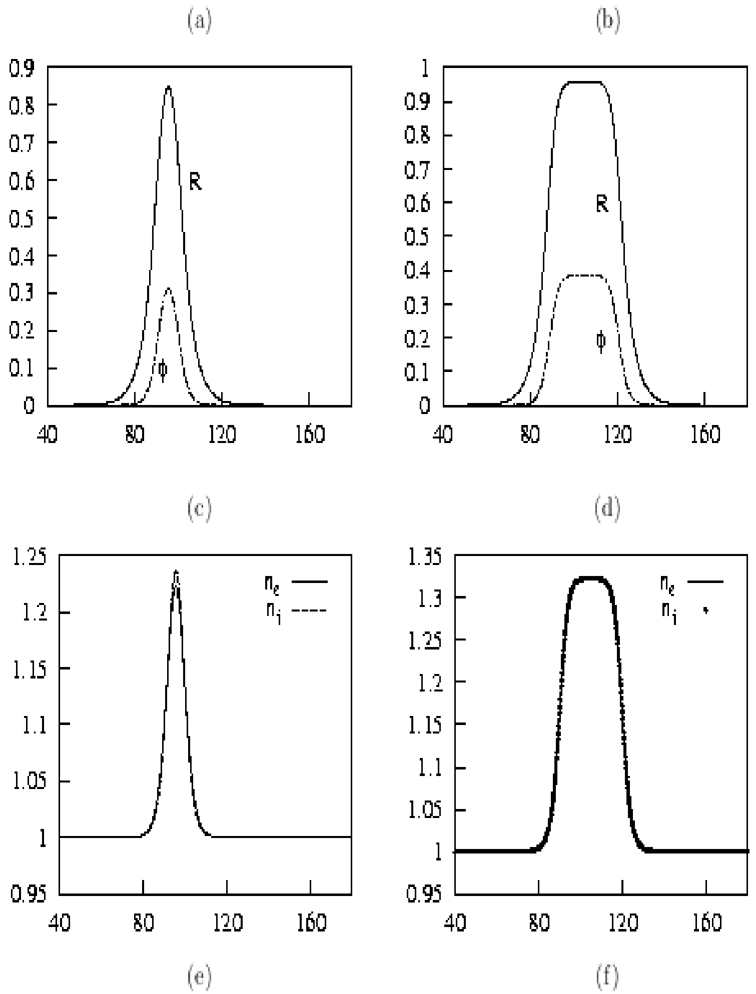
- No bright single peak solitons for $\beta < \sqrt{a}$ and solitons are dark in nature.(weak density perturabation case)
- Multi-peak solitons undergo bifurcation in their eigenvalue, a limit on eigenvalue and group velocity exist. (***No analytical understanding***)





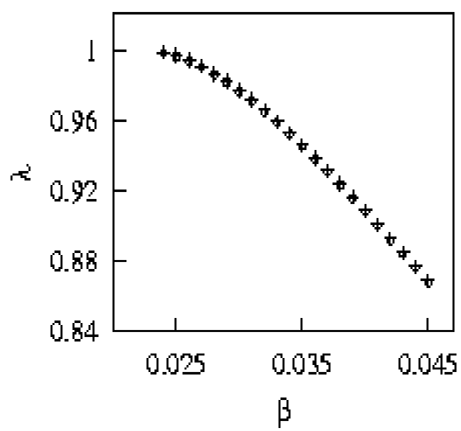
Peak p=2

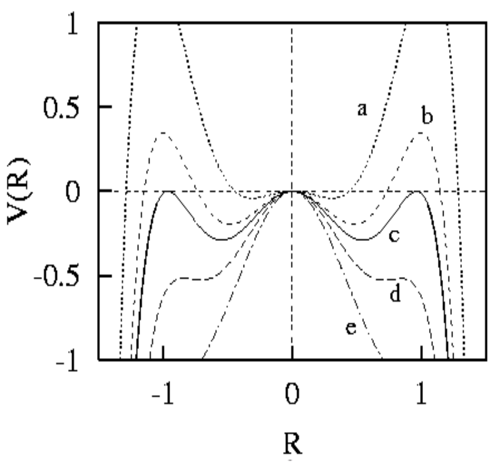
Finite amplitude solitons



$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - \frac{\beta}{\sqrt{1+R^2}}\frac{1}{\sqrt{\beta^2+2a[1-(1-\beta^2)\sqrt{1+R^2}]}}\right] = 0.$$

$$\lambda^2 = \frac{\beta^2\sqrt{1+2a}}{2aR_{max}^2}\left[1+\sqrt{1-\frac{4aR_{max}^2}{(1+2a)(1+R_{max}^2)}(1-\beta^2)}\right].$$



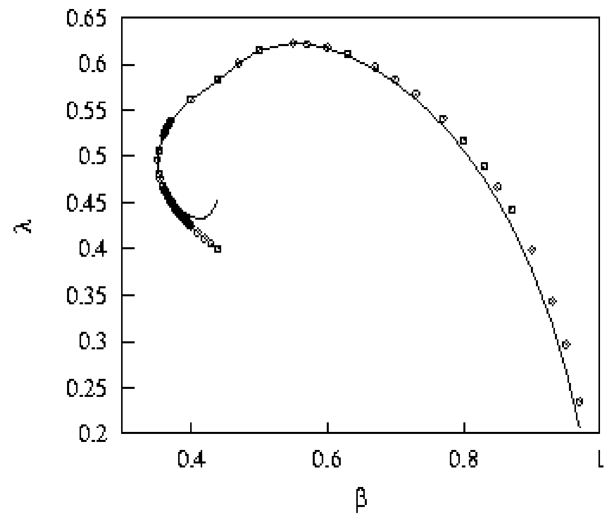


Multipeak case

In the central region of the pulse,

$$\phi'' = \frac{\beta}{1+\beta} - \frac{\beta}{\sqrt{\beta^2 - 2a\phi}},$$

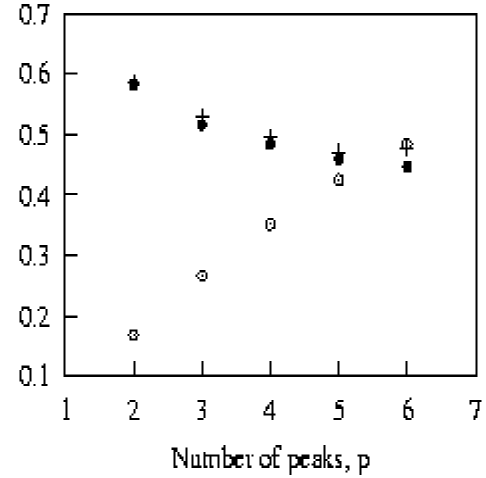
$$R'' = -\frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - a\right].$$



$$-\sqrt{\frac{a\beta}{(1+\beta)}}\xi_{max} = \left[-2a\phi_{max} + 2(1+\beta)\left(\beta - \sqrt{\beta^2 - 2a\phi_{max}}\right)\right]^{1/2}$$
$$+(1+\beta)\left[\sin^{-1}\left(\frac{1}{1+\beta-\sqrt{\beta^2-2a\phi_{max}}}\right) - \frac{\pi}{2}\right].$$

Eigenvalues

$$\lambda^2 = \left[\frac{\pi^2 p^2 \beta(1-\beta^2)}{8(1+\beta)}\frac{1}{\{f(\phi_{max})\}^2} + 1\right] a(1-\beta^2).$$



Main results

Single peak: Smooth transition occurs when ions are immobile, presence of finite amplitude.

When ion dynamics are taken into account, flat-top solitons exit.

Multi-peak solutions: Analytical description is provided by considering the standing wave pattern of the light wave in the central cavity.

The regime of validity of ion dynamics effects to be important is identified as

$$\sqrt{a} < \beta \leq \sqrt{2a\phi}$$

Is Cold Plasma model valid for very slowly propagating or stationary structures?

3. Warm plasma effects on slowly propagating solitons

- **Temperatures are non-relativistic,** $v_{the} \ll c \quad v_{thi} \ll c$
- **The equations of motion for electrons and ions are**

$$\left[\frac{\partial}{\partial t} + v_{e\|}\frac{\partial}{\partial x}\right](\gamma_e v_{e\|}) = \frac{\partial \phi}{\partial x} - \frac{1}{2\gamma_e}\frac{\partial}{\partial x}|A|^2 - \Gamma_e \alpha_e \frac{\partial}{\partial x}\log n_e,$$

$$\left[\frac{\partial}{\partial t} + v_{i\|}\frac{\partial}{\partial x}\right] v_{i\|} = -a\frac{\partial \phi}{\partial x} - a\Gamma_i \alpha_i \frac{\partial}{\partial x}\log n_i,$$

- **Other equations are the same as used in the cold plasma model.**

$$\xi = x - \beta t \text{ and } t = \tau. \qquad \gamma_{e0} = \sqrt{1 + R_0^2}$$

Reduced equations

$$\gamma_e(1 - \beta v_{e||}) - \phi + \Gamma_e \alpha_e \log n_e = \gamma_{e0},$$

$$\frac{v_{i||}^2}{2} - \beta v_{i||} + a(\phi + \Gamma_i \alpha_i \log n_i) = 0,$$

$$n_e = \frac{\beta}{\beta - v_{e||}},$$

$$n_i = \frac{\beta}{\beta - v_{i||}},$$

$$\phi'' = n_e - n_i,$$

$$R'' + \frac{R}{1 - \beta^2}\left[\frac{\lambda^2}{1 - \beta^2} - \frac{n_e}{\gamma_e}\right] = 0.$$

Quasineutral structures

$$n_e \approx n_i = n. \qquad v_{e||} \approx v_{i||} = v. \qquad \alpha = \Gamma_e \alpha_e + \Gamma_i \alpha_i$$

$$R'' - \left[1 + \frac{aR_0^2}{2(a\alpha - \beta^2)} - \frac{\lambda^2}{(1-\beta^2)}\right]\frac{R}{1-\beta^2} + \left[\frac{a(\alpha+1)-\beta^2}{a\alpha - \beta^2}\right]\frac{R^3}{2(1-\beta^2)} = 0.$$

$$R(\xi) = R_{max}\operatorname{sech}\left[\frac{\sqrt{1-\lambda^2-\beta^2}}{1-\beta^2}\right],$$

with the maximum amplitude R_{max} as

$$R_{max} = 2\sqrt{\frac{(a\alpha-\beta^2)(1-\beta^2-\lambda^2)}{(1-\beta^2)[a(1+\alpha)-\beta^2]}}.$$

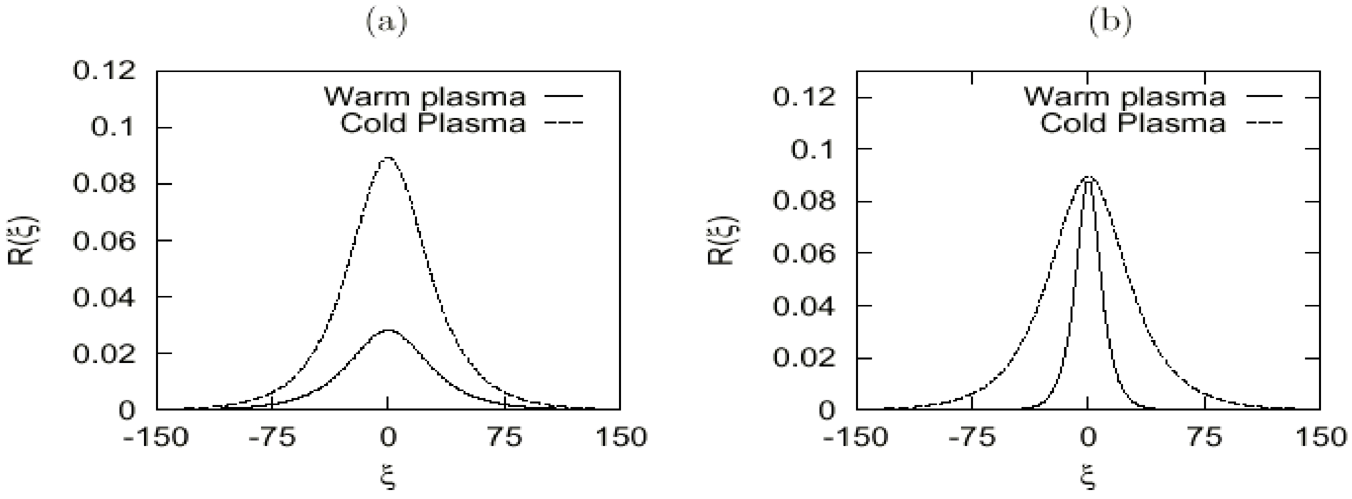
Bright solitons do not exist in the regime

$$a\alpha < \beta^2 < a(1+\alpha)$$

$$R(\xi) = 2\sqrt{\frac{\alpha(1-\lambda^2-\beta^2)}{(1-\beta^2)}}\,\mathrm{sech}\left(\frac{\sqrt{(1-\lambda^2-\beta^2)}}{(1-\beta^2)}\xi\right).$$

Cold plasma solitons $\beta = 0, \quad \lambda^2 - 1 \to 0.$

$$R(\xi) = 2\sqrt{1-\lambda^2}\,\frac{\cosh\left[\sqrt{1-\lambda^2}\xi\right]}{\cosh^2\left[\sqrt{1-\lambda^2}\xi\right] + \lambda^2 - 1}.$$



Propagation Speed β	Soliton Variety	Potential ϕ at center	Density n at center	Plasma Species involved
$0 \le \beta^2 < a\alpha_i$	bright	positive	evacuation	ions and electrons
$a\alpha_i < \beta^2 < a\alpha$	bright	negative	evacuation	ions and electrons
$a\alpha < \beta^2 < a(1+\alpha)$	dark	negative	evacuation	ions and electrons
$a(1+\alpha) < \beta^2$	bright	positive	accumulation	ions and electrons
$a(1+\alpha) \ll \beta^2$	bright	positive	evacuation	only electrons

Table 3.1: Table I showing the characteristics of the small amplitude solitons in different group velocity regime Various symbols used are $a = m_e/m_i$, $\alpha_{e,i} = T_{e,i}/m_e c^2$, $\alpha = \Gamma_e \alpha_e + \Gamma_i \alpha_i$, $\Gamma_e = 1$, $\Gamma_i =$ ion adiabaticity parameter, $\beta =$ normalized propagation speed

4. Warm plasma effects on arbitrary amplitude solitons

- **Full set of equations are considered.**
- **Ions are cold, which is valid when** $\beta^2 > a\alpha_i$
- **Subsonic solitons are considered in the regime,** $\beta < \sqrt{a\alpha_e} < \sqrt{a}$
- **Velocities are smaller therefore** $\gamma_e \approx \sqrt{1 + R^2}.$
- **Electron and ion densities are,**

$$n_e = \exp\left[\frac{1 - \sqrt{1 + R^2}}{\alpha_e}\right]. \qquad n_i = \frac{\beta}{\sqrt{\beta^2 - a^2 R^2 - 2a\phi}}.$$

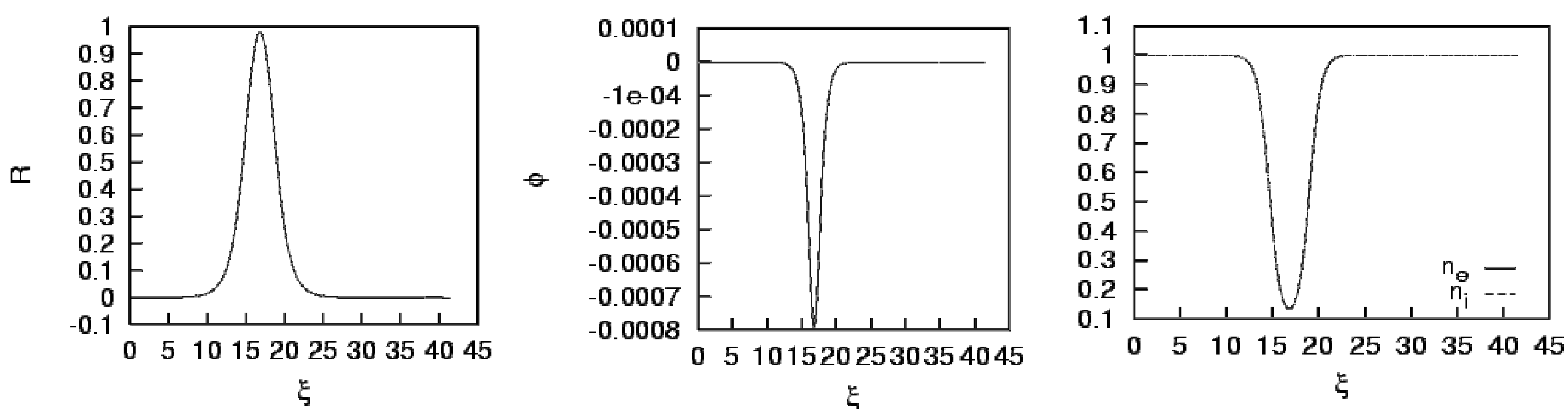
Single peak solitons

Light wave equation is

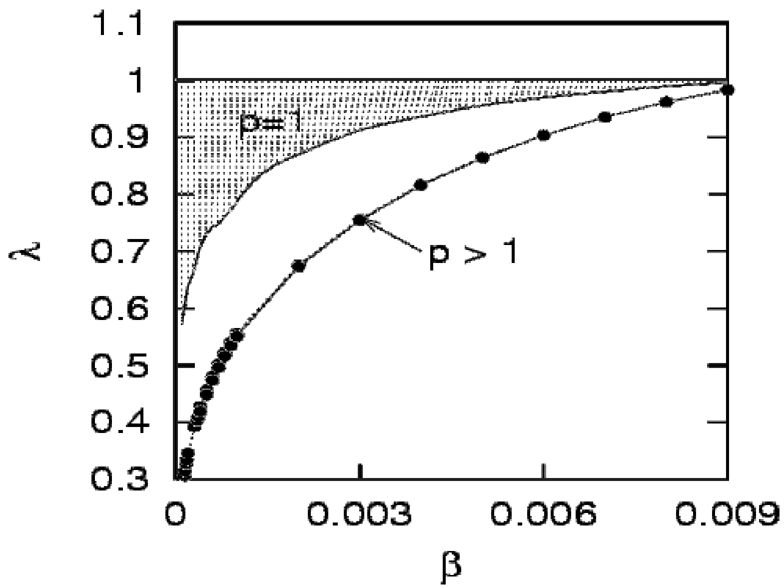
$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - \frac{1}{\sqrt{1+R^2}}\exp\left(\frac{1-\sqrt{1+R^2}}{\alpha_e}\right)\right] = 0.$$

Eigenvalue is

$$\lambda^2 = \frac{2\alpha_e(1-\beta^2)}{R_{max}^2}\left[1 - \exp\left(\frac{1-\sqrt{1+R_{max}^2}}{\alpha_e}\right)\right]$$

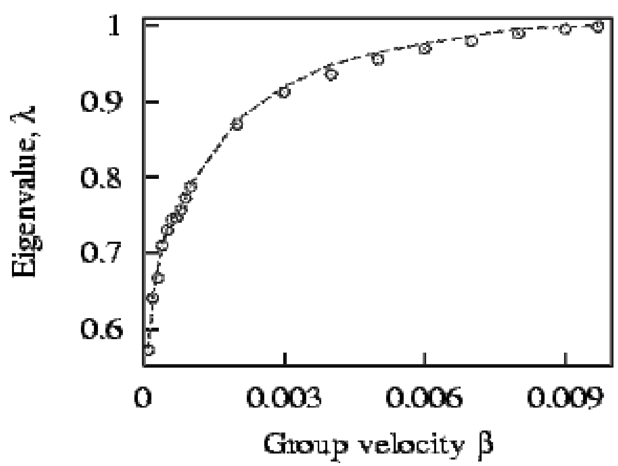


Eigenvalue spectrum

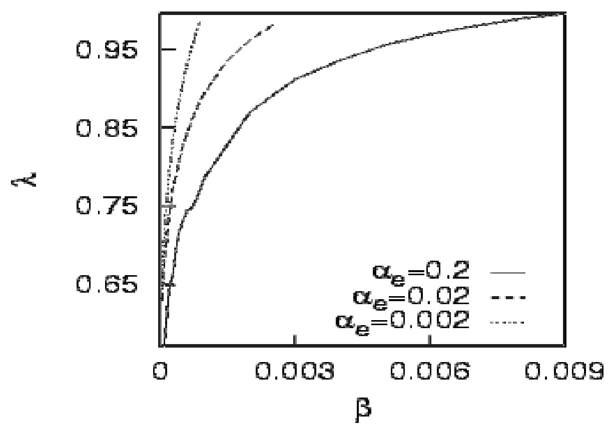


$$\alpha_e = 0.2,$$

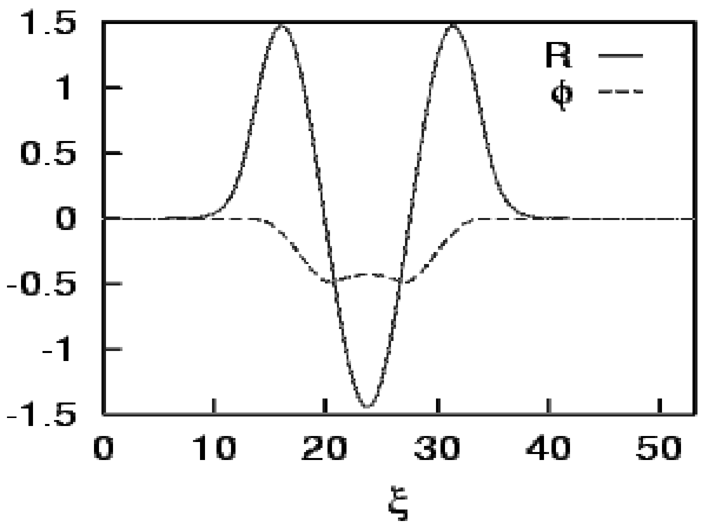
$$\Gamma_e = 1$$

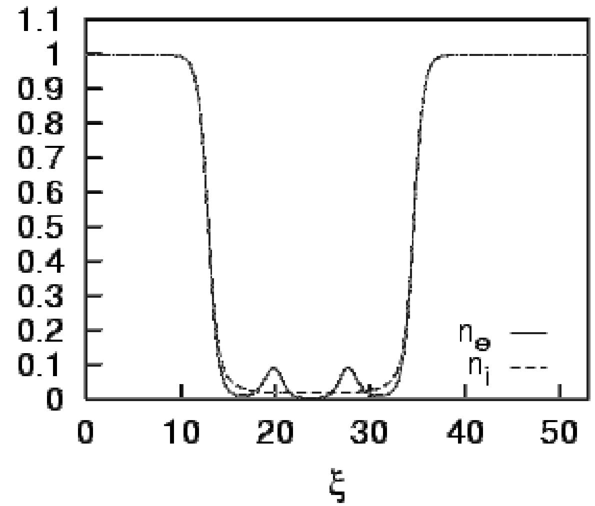


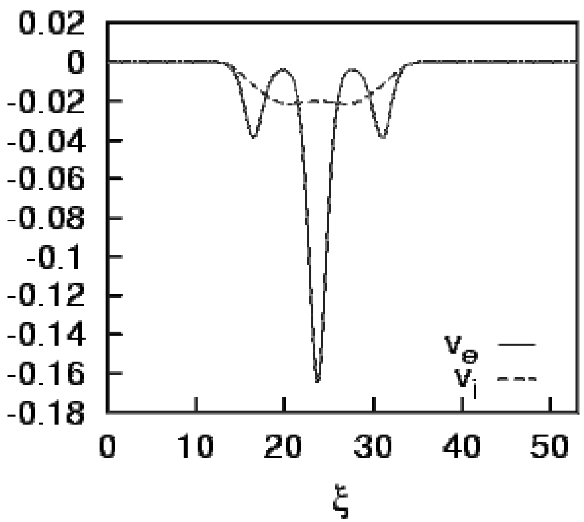
Variation with temperature



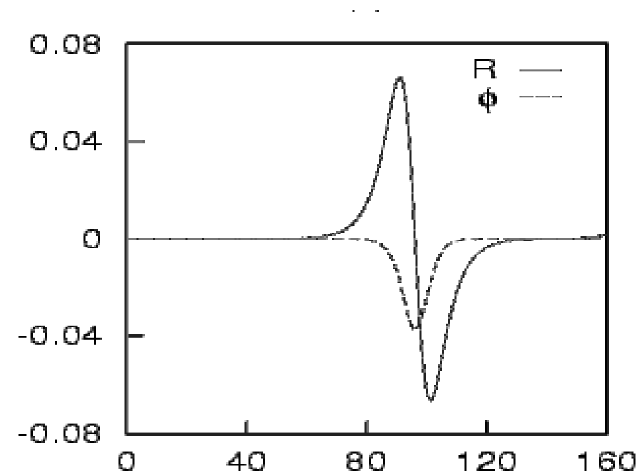
$$\beta = 0.0001, \quad \alpha_e = 0.2, \quad \Gamma_e = 1$$







$$\beta \to \sqrt{a\alpha_e}$$



$$\Psi = \bar{R}/\Lambda\alpha_e,\ \Phi = \bar{\phi}/\Lambda\alpha_e \text{ and } \eta = \sqrt{\Lambda}\xi,$$

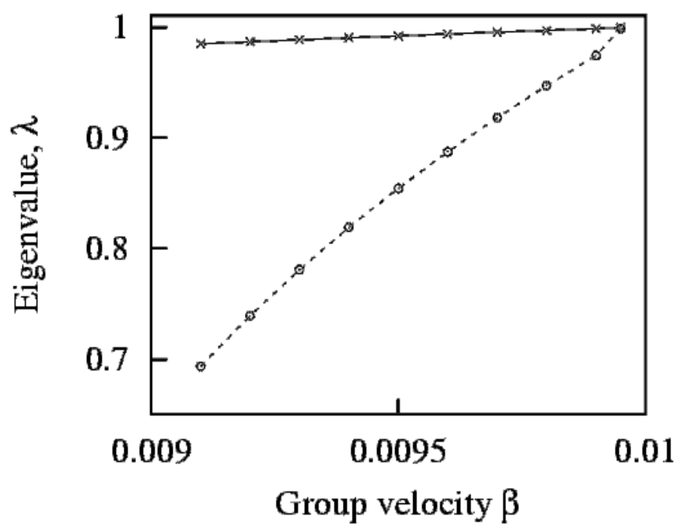
$$\Phi_{\eta\eta} + \frac{(a\alpha_e - \beta^2)}{\Lambda\beta^2}\Phi = \Psi^2,$$

$$\Psi_{\eta\eta} - \Psi(1-\Phi) = -\Psi^3.$$

When $\Psi^3 \to 0,$

$$\Psi(\eta) = 6\sqrt{2}\operatorname{sech}(\eta)\tanh(\eta),$$

$$\Phi(\eta) = \pm 6\operatorname{sech}^2(\eta).$$



$$\lambda^2 = \left[1 - \frac{a\alpha_e - \beta^2}{2\beta^2\alpha_e}(1-\beta^2)\right](1-\beta^2).$$

- **Warm plasma effects give raise to new regime for bright solitons below ion acoustic speed.**

- **(These were not obtained in the earlier studies)**

  (Yu et al, 1978, Rao et al, 1983)

- **Different regions are delineated for the quasineutral case**

- **(Multi-peak solitons and their spectral characters are obtained, these were not known earlier)**

- **Warm plasma solitons arise due balancing of Ponderomotive force of the light field by the pressure gradient force.**

5. Solitons in an Electron-positron-ion plasma.

- **Relevant in astrophysical context where presence of ions impurities are inevitable in an electron-positron plasma.**

  (pure electron-positron plasma do not support bright solitons)

- **A new parameter defining the ratio of ion density to electron density in the equilibrium composition is important in determining the characteristics of the solutions.**

- **Earlier investigations have considered only single peak structures.**

  (Rizzato, 1988, Berezhiani et al, 1994)

- **Conditions on the existence of these entites have not been obtained.**

- **Spectral characteristic have not been studied in the past.**

Model equations

$$\xi = x - \beta t \text{ and } t = \tau. \qquad n_{p0}/n_{e0} = 1 - \eta.$$

$$\gamma_e(1 - \beta v_{e||}) - \phi = 1,$$
$$\gamma_p(1 - \beta v_{p||}) + \phi = 1,$$
$$\gamma_i(1 - \beta v_{i||}) + a\phi = 1,$$

$$n_e = \frac{\beta}{\beta - v_{e||}},$$
$$n_p = \frac{\beta}{\beta - v_{p||}},$$
$$n_i = \frac{\beta}{\beta - v_{i||}},$$

$$\phi'' = n_e - (1 - \eta)n_p - \eta n_i,$$

$$R'' + \frac{R}{1 - \beta^2}\left[\frac{\lambda^2}{1 - \beta^2} - \frac{n_e}{\gamma_e} - (1 - \eta)\frac{n_p}{\gamma_p} - a\eta\frac{n_i}{\gamma_i}\right] = 0.$$

- **For small amplitudes,** $R^2 \ll 1$ $\sqrt{1+R^2} \approx 1 + R^2/2.$

$$\phi'' + \frac{2-\eta}{\beta^2}\phi = \frac{\eta}{2\beta^2}R^2,$$

$$R'' + \frac{R}{1-\beta^2}\left[\frac{\lambda^2}{1-\beta^2} - (2-\eta) + \frac{\eta}{\beta^2}\phi\right] = \frac{2-\eta}{2\beta^2}R^3.$$

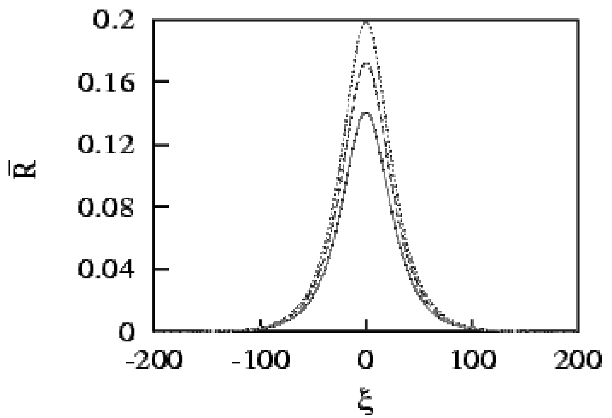
$$\bar{R} = \eta R/\beta\sqrt{2(1-\beta^2)}), \; \bar{\phi} = \eta\phi/\beta^2(1-\beta^2)$$

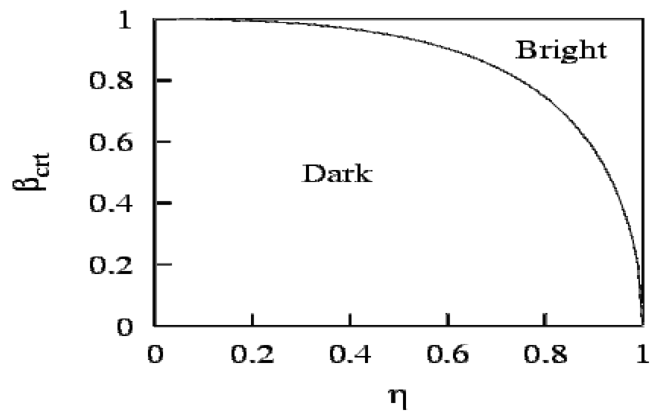
$$\bar{\phi}'' + \frac{2-\eta}{\beta^2}\bar{\phi} = \frac{\bar{R}^2}{\beta^2},$$

$$\bar{R}'' + (\bar{\phi} - \Lambda)\bar{R} = \frac{2-\eta}{\eta^2}(1-\beta^2)\bar{R}^3,$$

Single peak

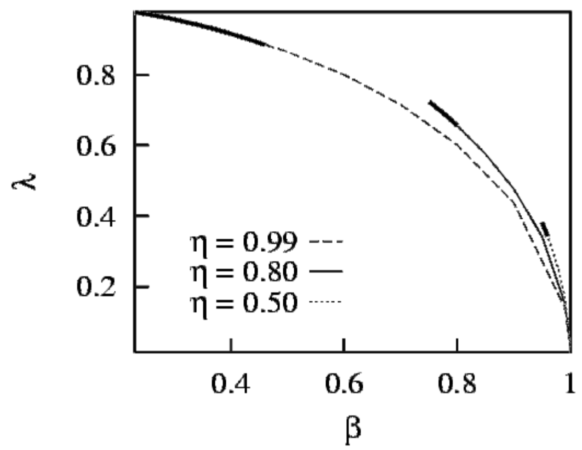
$$\bar{R}(\xi) = \eta\sqrt{\frac{2\Lambda(2-\eta)}{4(\eta-1)+\beta^2(2-\eta)^2}}\ \mathrm{sech}\left(\frac{\sqrt{\Lambda}}{2}\xi\right)$$





For smaller values of η

The solitons have higher amplitudes



$$\beta_{crt} = 2\sqrt{1-\eta}/(2-\eta).$$

Multi-peak structures

$\beta \rightarrow 1$ limit,

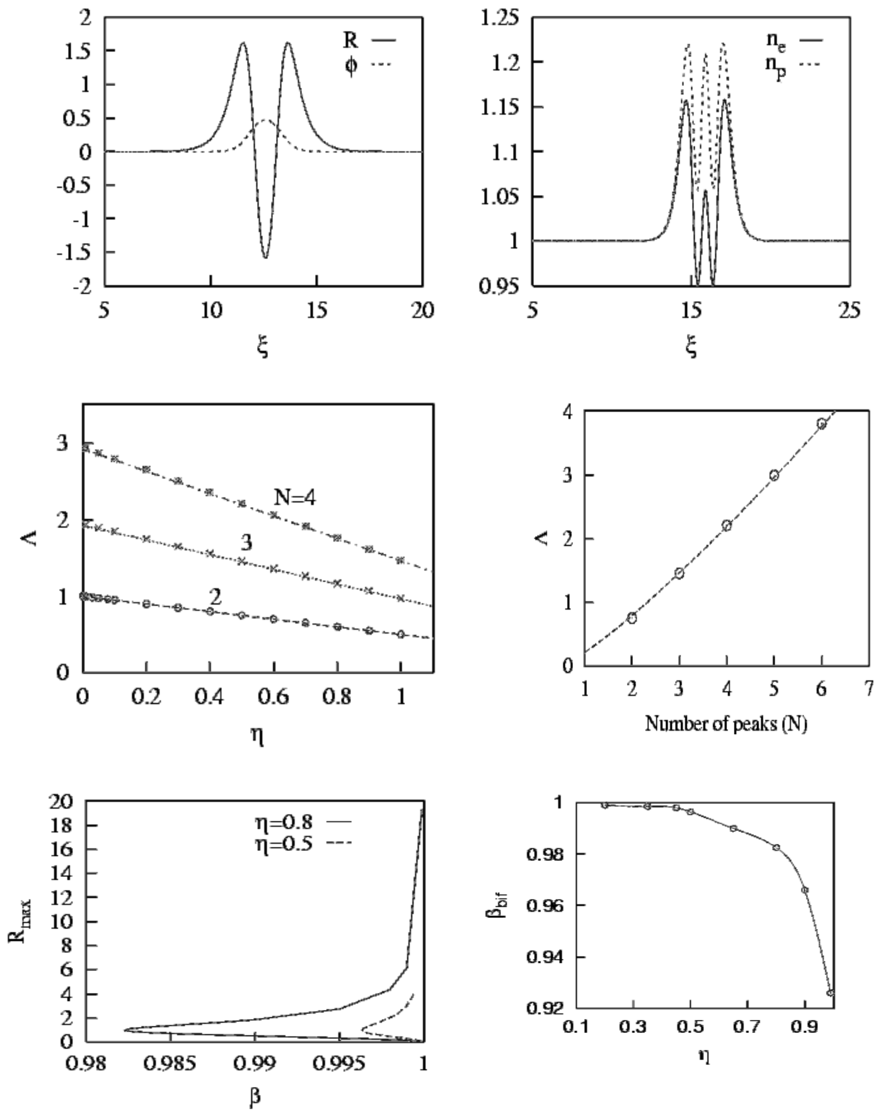
$$\bar{\phi}'' + (2-\eta)\bar{\phi} = \bar{R}^2,$$
$$\bar{R}'' + (\bar{\phi} - \Lambda)\bar{R} = 0.$$

WKB solution in the oscillating region

$$\bar{R} = C_N \psi^{-1/4} \sin\left[\int_0^{\xi} \sqrt{\psi} d\xi - N\pi/2\right]$$

Eigenvalue is

$$\Lambda \approx 0.197(2-\eta)\left(N - \frac{1}{2}\right)^{6/5}$$



6. New classes of solitons

- Several new classes of solitons are possibile.

- Model equations have similarities with equations in other systems (solitons in optical waveguides).

- Cold Plasma model is being used.

Quasi-solitons

- **Linear radiation arise due to resonance interaction.**
- **For weak amplitudes,** $R^2 \ll 1,\ \phi \ll 1$
- **The model equations are:**

$$\Lambda = (1 - \lambda^2 - \beta^2)/(1 - \beta^2)^2;$$

$$\bar{\phi}'' + \frac{\bar{\phi}}{\beta^2} = \frac{\bar{R}^2}{2\beta^2},$$

$$\bar{R}'' + \bar{R}\left(\frac{\bar{\phi}}{\beta^2} - \Lambda\right) = \frac{1-\beta^2}{2\beta^2}\bar{R}^3.$$

$$\bar{R} = R/\sqrt{(1-\beta^2)}.$$

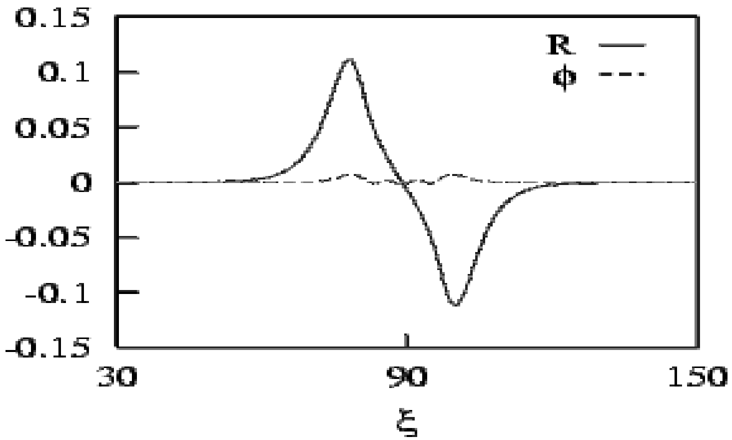
$$\bar{\phi} = \phi/(1-\beta^2)$$

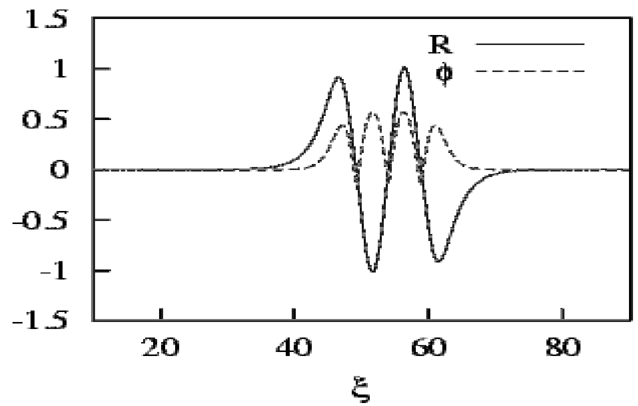
- **The space charge potential is**

$$\bar{\phi} = \frac{\bar{R}_0^2}{2\beta}\sin\left(\frac{\xi}{\beta}\right)\int_{-\infty}^{\xi}\cos\left(\frac{\eta}{\beta}\right)\operatorname{sech}^2\left(\frac{\bar{R}_0}{\sqrt{2}}\xi\right)d\eta - \frac{\bar{R}_0^2}{2\beta}\cos\left(\frac{\xi}{\beta}\right)\int_{-\infty}^{\xi}\sin\left(\frac{\eta}{\beta}\right)\operatorname{sech}^2\left(\frac{\bar{R}_0}{\sqrt{2}}\xi\right)d\eta - \frac{\pi}{2\beta^2}\frac{1}{\sinh\left(\frac{\pi}{\bar{R}_0\beta\sqrt{2}}\right)}\sin\left(\frac{\xi}{\beta}\right).$$

Embedded Soltions

- Linear radiation is trapped by two solitons.
- Radiations at infinity undergo destructive interference.



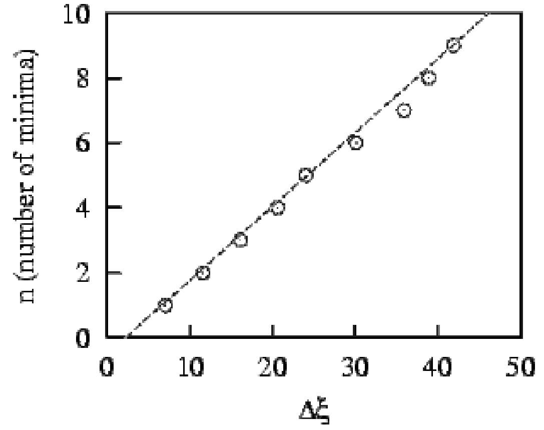


$$\bar{R}_1 = \bar{R}_0 \operatorname{sech}\left[\frac{\bar{R}_0}{\sqrt{2}}(\xi - \xi_a)\right] e^{i\theta_1},$$

$$\bar{\phi}_1 = \bar{R}_0^2 \operatorname{sech}^2\left[\frac{\bar{R}_0}{\sqrt{2}}(\xi - \xi_a)\right] e^{2i\theta_1} + \bar{\phi}_l \sin\left(\frac{\xi - \xi_a}{\beta} + 2\theta_1\right),$$

$$\bar{R}_2 = \bar{R}_0 \mathrm{sech}\left[\frac{\bar{R}_0}{\sqrt{2}}(\xi - \xi_b)\right] e^{i\theta_2},$$

$$\bar{\phi}_2 = \bar{R}_0^2 \mathrm{sech}^2\left[\frac{\bar{R}_0}{\sqrt{2}}(\xi - \xi_b)\right] e^{2i\theta_2} + \bar{\phi}_l \sin\left(\frac{\xi - \xi_b}{\beta} + 2\theta_2\right).$$



Interference pattern at infinity:

$$|\bar{\phi}_{tot}|^2 = 2\bar{\phi}_l\left[1 + \cos\left(\frac{\Delta\xi}{\beta} - 2\Delta\theta\right)\right].$$

$$n = (\Delta\xi/\pi\beta - 1)/2$$

Destructive interference occurs when

Phase difference

$$\Delta\theta = 0, \pi; \qquad \Delta\xi = (2n+1)\pi\beta$$

$$\Delta\theta = \theta_1 - \theta_2.$$

Path difference

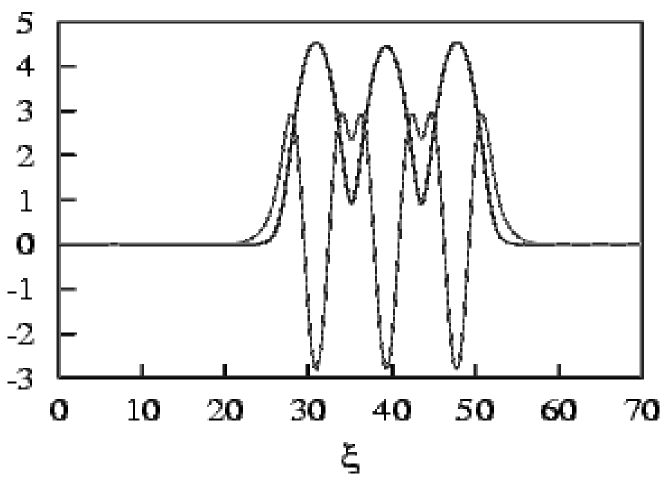
Or,

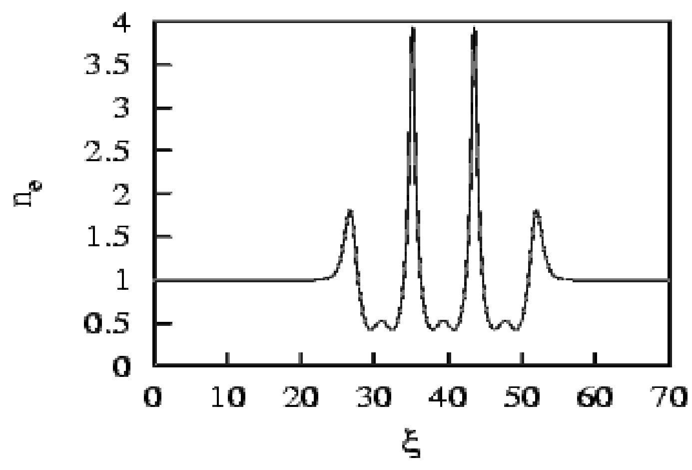
$$\Delta\theta = \frac{\pi}{2}, \frac{3\pi}{2}; \qquad \Delta\xi = 2n\pi\beta.$$

$$\Delta\xi = \xi_a - \xi_b$$

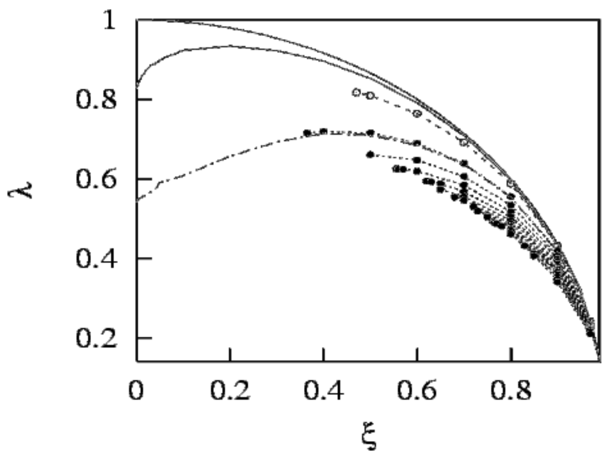
Multi-hump solitons

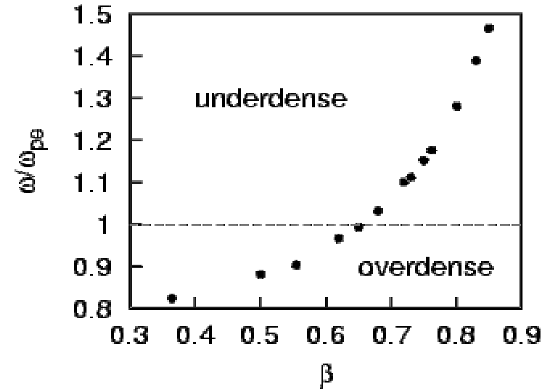
- Vector potential has multiple peaks, scalar potential has multiple humps.
- Each hump of the scalar potential encloses many peaks of the vector potential.
- Electron density has alternating regions of humps and dips thus resembles a Bragg's gratting.

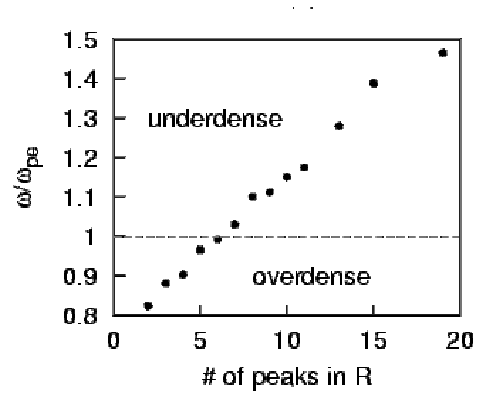


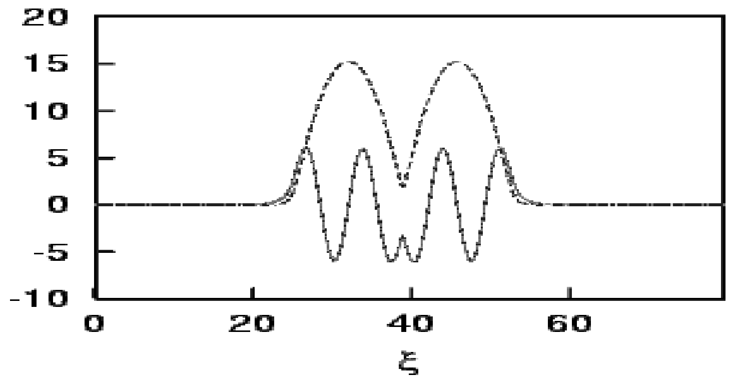


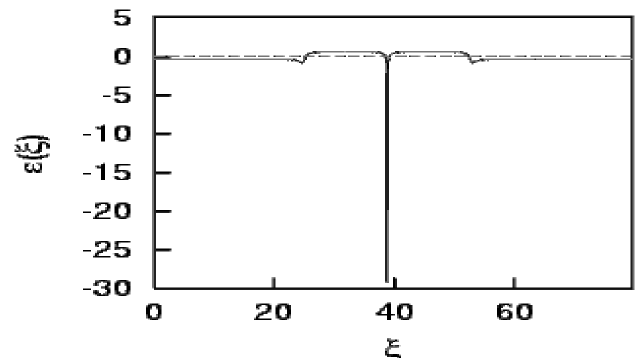
Eigenvalue specturm and soliton breaking

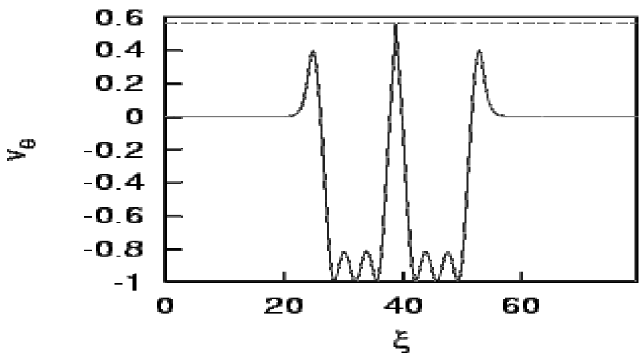


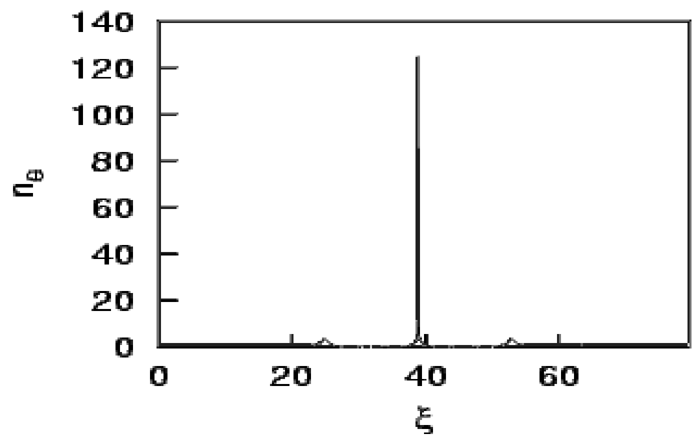












7. Conclusions

- Questions related to single peak solitons in a cold plasma are addressed.
- Analytical description for multi-peak solitons are provided.
- Effect of finite temperature on slow speed entities provides a new

  regime of propagation speed for bright solitons.
- Multi-peak solitons are identifed in the e-p-I admixture plasma.

  Eigenvalue condition, limit on the propagation speed are derived.
- Three new classes of solitons are obtained.

Physical mechanism :

Cold plasma: Balancing of ponderomotive force and the force due to space charge field.

Warm plasma : Balancing of ponderomotive force due to pressure gradient forces.

Still More !!!

- **Stability analysis of these coherent structures.**
- **Self generated magnetic field effects, and plasma inhomogenity effects.**
- **Interaction mechanism of several solitons and the processes involved in the formation of soliton gas, soliton necklace etc.**
- **Transition from coherent to turbulant state.**
- **Extension to higher dimensions.**